STOCHASTIC MODEL OF THE PROCESS OF STARTING PISTON COMBUSTION ENGINES AND THE PRACTICAL USE OF THE PROCESS

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Abstract

The paper presents a proposal of a model of diesel engine starting process as semi-Markov process. States of the process are the following: cold engine state (s_1), warm engine state (s_2) and hot engine state (s_3). Idea of using the model for determining a quantity of harmful substances emitted in exhaust gases during starting of engine, has also been proposed herein.

The paper provides a possibility of considering in researches on the quantity of harmful substances contained in exhaust gases, a mass of harmful substances as a random variable. It has been accepted that statistics of the random variable has distribution asymptotically normal. The stochastic model of self-ignition engines starting process enables deriving the limiting distribution of the process, being the probabilities of occurring the mentioned states (s_1), (s_2) and (s_3) one by one. The distribution is indispensable to determine the expected value of the mass of harmful substances emitted in time of successive re-starts of engine performed during long operation. The second approach to estimation of the mentioned mass considered as a random variable enables estimating its expected value in time of long-term work of engines till the first failure or in the period between two successive preventive maintenances. The paper proposes the interval estimation because makes it possible to assess accuracy of the estimation. In this case the mentioned value is determined in the form of interval with random edges (limits), which comprises unknown value of the mass of harmful substances emitted by engine, at the probability called confidence level.

Keywords: *transport, diesel engine, semi-Markov process, toxic substance, modelling*

MODEL STOCHASTYCZNY PROCESU URUCHAMIANIA TŁOKOWYCH SILNIKÓW SPALINOWYCH I JEGO PRAKTYCZNE ZASTOSOWANIE

Streszczenie

Zaproponowano model procesu rozruchu silników o zapłonie samoczynnym, w formie procesu semimarkowskiego, dyskretnego w stanach i ciągłego w czasie eksploatacji. Stanami tego procesu są: stan zimny silnika (s_1), stan ciepły silnika (s_2) i stan gorący silnika (s_3). Zaproponowano również koncepcję zastosowania opracowanego modelu do określenia ilości substancji szkodliwych, jakie emitowane są w spalinach podczas rozruchu silnika. Przedstawiono także możliwość uwzględnienia, w badaniach ilości substancji szkodliwych zawartych w spalinach, masy substancji zanieczyszczonej jako zmiennej losowej. W rozważaniach tych przyjęto, że statystyka tej zmiennej losowej ma rozkład asymptotycznie normalny. Przedstawiony model stochastyczny procesu rozruchu silnika o zapłonie samoczynnym umożliwia wyznaczenie rozkładu granicznego tego procesu, którym są prawdopodobieństwa pojawiania się kolejno wspomnianych stanów: (s_1), (s_2) i (s_3). Rozkład ten jest niezbędny do określenia wartości oczekiwanej masy substancji szkodliwych emitowanej podczas kolejnych rozruchów silnika, dokonywanych w czasie długotrwałej ich eksploatacji. Drugie podejście do oszacowania wspomnianej masy substancji szkodliwych, rozumianej jako zmiennej losowej, umożliwia oszacowanie jej wartości oczekiwanej w czasie długotrwałej pracy silników do pierwszego uszkodzenia bądź w okresie między dwoma kolejnymi ich obsługami profilaktycznymi. Oszacowanie wartości oczekiwanej masy substancji szkodliwych emitowanej podczas długotrwałej pracy silników może być dokonane przez zastosowanie estymacji punktowej lub przedziałowej. W opracowaniu zaproponowano estymację przedziałową, ponieważ umożliwia

ona określeni dokładności tego oszacowania. W tym przypadku, wspomniana wartość jest określana w formie przedziału o losowych końcach (granicach), w który zawarta jest nieznana wartość masy emitowanych przez silnik substancji szkodliwych, z prawdopodobieństwem nazywanym poziomem ufności.

Słowa kluczowe: *silnik o zapłonie samoczynnym, proces semi-Markova, substancja toksyczna*

1. Introduction

Considerations in the phases of combustion engines designing and operating include not only parameters characterizing the energy conversion, like [2, 14, 15, 17, 18]: general ability of engine, heat emission rate, heat usage rate, heat evolution rate, pressure escalation rate, etc. Emission content should also be submitted to analysis for such toxic compounds as carbon monoxide (CO), hydrocarbons (C_nH_m), nitric oxide (NO_x), particulate solid (PM- Particulate Matte) and sulfur compounds (SiO_2, SiO_3, H_2SO_3, H_2SO_4), aldehydes and others [1, 12]. From researches on fuel combustion processes results that physical (not chemical) processes have great influence on the quantity of emission of particular toxic compounds included in exhaust gases. The other issue from the researches is that their emission is closely connected with each other and possibility of reduction of one of them can cause emission increase of another one. Therefore, any action to reduce the content of toxic compounds in exhaust gases must be followed by a compromise ensuring minimization of harmful impact of toxic exhaust compounds on natural environment.

Emission of the toxic compounds depends, among others, on engine thermal state, especially while starting engine. Thus, this paper provides a proposal of an original method to determine the mass of toxic compounds emitted by self-ignition or spark-ignition engine during starting. The method considers the stochastic model of starting engines of this kind. It has been shown that the model can be presented in the form of semi-Markov process.

Professional literature proves that the most toxic compounds are formed in exhaust gases during so called "cold engine starting", so when the engine is started after not sufficient pre-warming phase [1, 9, 11, 19]. From this reason there is a need to create a model of engine starting process considering at least three states of engine: cold, warm (warmed up) and hot.

2. Model of self-ignition engine starting process

Considering the process of starting a self-ignition engine, at least three thermal states of it can be distinguished, that exist directly before starting, i.e. *cold state* (s_1), *warm state* (s_2), and *hot state* (s_3). The states may be values of the process $\{U(t): t \in T\}$ being a real model of the engine starting process. The model can be presented in the form of semi-Markov process having the set of states [5]:

$$S = \{s_1, s_2, s_3\} \tag{1}$$

with the following interpretation of the states:

- s_1 cold state which enables starting engine in ambient conditions (conditions of ship power plant) with typical low temperatures (T) not higher than 290 K;
- s_2 warm state which enables starting engine in conditions of pre-warming (T > 300 K) performed on engine in state s_1 before starting;
- s_3 hot state which enables starting engine in its operation (work) conditions following from the need of stopping the engine work under even large load and then re-starting.

Thus, the process is a three-state process with continuous realizations (process continuous in time). This is a semi-Markov process [4, 5, 6, 7] and, as a model of changes of the states of self-ignition engine starting, is the simplest model which may be of practical significance.

In this relation the set of states of starting self-ignition engines $S = \{s_1, s_2, s_3\}$ can be considered as the set of values of the stochastic process $\{U(t): t \in T\}$ with constant intervals and right-side continuous realizations.

Initial distribution of the process $\{U(t): t \in T\}$ of any self-ignition or spark-ignition engine is defined by the formula [5]:

$$P_i = P\{U(0) = s_i\} = \begin{cases} 1 & dla \quad i = 1 \\ & \\ 0 & dla \quad i = 2,3 \end{cases}. \tag{2}$$

Functional matrix of the process is as follows:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & Q_{32}(t) & 0 \end{bmatrix}. \tag{3}$$

Thus, for the presented process $\{U(t): t \in T\}$ with functional matrix expressed by the formula (3), the following limiting distribution can be determined:

$$P_1 = \frac{\pi_1 E(T_1)}{H}, \quad P_2 = \frac{\pi_2 E(T_2)}{H}, \quad P_3 = \frac{\pi_3 E(T_3)}{H} \tag{4}$$

at:

$$\pi_1 = \frac{p_{31} + p_{12}p_{32}}{2 + p_{12}p_{23}p_{31} + p_{13}p_{21}p_{32}}, \quad \pi_2 = \frac{p_{32} + p_{12}p_{31}}{2 + p_{12}p_{23}p_{31} + p_{13}p_{21}p_{32}},$$

$$\pi_3 = \frac{1 - p_{12}p_{21}}{2 + p_{12}p_{23}p_{31} + p_{13}p_{21}p_{32}},$$

$$H = \pi_1 E(T_1) + \pi_2 E(T_2) + \pi_3 E(T_3),$$

where:

P_1, P_2, P_3 – probability that combustion engine (self-ignition or spark-ignition) is started from the states accordingly: s_1, s_2, s_3,

π_j – limiting probability of the Markov chain inserted in the process $\{U(t): t \in T\}$, describing possibility of occurring the state s_j, $j = 1, 2, 3$,

p_{ij} – probability of the process $\{U(t): t \in T\}$ transition from the state s_i to the state s_j;

$E(T_j)$ – expected value of duration of the state s_j, ($j = 1, 2, 3$).

The presented states of starting any self-ignition or spark-ignition engine are connected with particular thermal states of the engine. They reflect states existing in practice. Therefore, the proposed model can be employed for determining the mass of toxic compounds.

3. Determination of toxic compounds mass during phase of engine starting.

Analysis of research results on the combustion process run, shows that during combustion there are different conditions for toxic compounds to form in emission.. The most significant of them are: carbon monoxide (CO), hydrocarbons (C_nH_m), nitric oxide (NO_x), particulate solid and substances in very little amounts (forming low concentrations) like sulfur compounds (SiO_2, SiO_3,

H_2SO_3 and H_2SO_4), aldehydes and others [1, 12, 14, 18]. In practice there is a need to control emission of the mentioned substances.

Emission of toxic substances can be determined from the following formula [1]:

$$e_k = \frac{v_{mix}\rho_k K_H c_k}{s} 10^{-6} \tag{5}$$

at $k = 1, 2, ..., n$ – type (number) of the given toxic substance,

where:

e_k – mass of toxic substance [g/km],
v_{mix} – volume of dilute emission in normal conditions [dm^3/test],
ρ_k – density of toxic substance in normal conditions [g/dm^3],
K_M – correction rate of humidity of nitric oxide mass,
c_k – concentration of the substance toxic for surroundings [ppm],
s – rout traveled by a transport mean (car, sea-going ship) during testing [km].

Emission e_k ($k = 1, 2, ..., n$) is different depending on whether the starting of engine runs from the cold state (s_1), warm state (s_2) or hot state (s_3). During test performance, on the rout (s) traveled by the given transport mean (car, sea-going ship), engine can be started several times from its different states mentioned above. The emission will be also different for the engine depending on technical state of the fuel system and fuel quality. Ambient conditions in which the engine is started are important, as well. Therefore, the value of toxic substances emission determined from the formula (5) may be accepted as emission realization considered as a random variable. Hence, taking into account the dependence (4), the mass of toxic substances can be expressed by the following formula:

$$\mathrm{E}(E_k) = \frac{(p_{31} + p_{12}p_{32})E(T_1)}{M} e_{1k} + \frac{(p_{32} + p_{12}p_{31}E(T_2)}{M} e_{2k} + \frac{(1 - p_{12}p_{21})E(T_3)}{M} e_{3k} \tag{6}$$

at:

$$M = E(T_1) + p_{12}E(T_2) + (1 - p_{12}p_{23})E(T_3),$$

where:

e_k – mass of the substance toxic for the natural environment ($k = 1, 2, ..., n$),
p_{ij} – probability of transition of the engine starting process $\{U(t): t \in T\}$ from the state s_i to the state s_j, $i \neq j$; $i, j = 1, 2, 3$,
$E(T_j)$ – expected value of state s_j ($j = 1, 2, 3$) duration.

The formula (6) follows from consideration of all dependences provided in the formula (4) which determines the probabilities P_1, P_2 i P_3.

The mentioned toxic substances contained in emission like: carbon monoxide (CO), nitric oxides (NO_x), the most of all NO, hydrocarbons (C_nH_m), particulate solid (PM) and sulfur compounds, as sulfur dioxide (SO_2), sulfur trioxide (SO_3), sulfurous acid (H_2SO_3), sulfuric acid H_2SO_4), can be and should be considered as random variables. That is because these characteristics are such variables which, in the result of successive measures, take different numerical values with determined probability. So, the mass e_k of the mentioned toxic substances in emission, calculated from the formula (5), can be considered as realization of random variable E_k, where k = CO, NO,

C_nH_m, PM, SO_2, SO_3, H_2SO_3, H_2SO_4. That means that determined probability is assigned to each possible value of any random variable E_k,. This fact can be described in the following form [3, 13]:

$$P(E_k = e_{ki}) = p_{ki} \ (i = 1, 2, ..., n). \tag{7}$$

The formula shows that probability p_{ki} is a function of the values which can be taken by a random variable E_k. So, for any random variable E_k the following description can be made:

$$p_{ki} = f(e_{ki}). \tag{8}$$

The characteristic of the function is that the sum of probabilities determined by the dependence (8) is equal to 1, what can be described as follows:

$$\sum_{i=1}^{n} p(e_{ki}) = 1 . \tag{9}$$

Analyzing toxic substances contained in the emission as random variables E_k and making the successive measures of their mass value, the average mass value (arithmetic average) can be determined from the formula [3, 8, 10, 16]:

$$\bar{e}_k = \frac{1}{n}\sum_{i=1}^{n} e_{ki} , \tag{10}$$

where:

e_{ki} – value of the analyzed characteristic (random variable) E_k .

Average value $\bar{e}_k$, determined from the formula (10) is the observed value of statistics $\overline{E}_k$. The statistics is a random variable [3]

$$\overline{E}_{kn} = \frac{1}{n}\sum_{i=1}^{n} E_{ki} , \tag{11}$$

where E_{ki} – random variable with the same (arbitrary) distribution of a common expected value $E(E_{ki}) = m_{1k}$ and variation $D^2(E_{ki}) = \sigma_k^2 \neq 0$.

Thus, random variable $\overline{E}_k$ is an arithmetic average with n independent random variables E_{ki} of identical distribution. Expected value and variation of random variable $\overline{E}_k$ are defined in the form of dependences [3, 8]:

$$E(\overline{E}_{kn}) = E(E_{ki}) = m_{1k} , \qquad D^2(E_{kn}) = \frac{1}{n} D^2(E_{ki}) = \frac{\delta_k^2}{n} . \tag{12}$$

From the Lindeberg-Levy theorem follows [3], that the random variable $\overline{E}_k$ (statistics) has got a distribution asymptotically normal $N(m_{1k}, \frac{\sigma_k}{\sqrt{n}})$ regardless of the character of the random variable E_k. That means that the arithmetic average with n independent random variables E_{ki},

arbitrary but identical distribution, common expected value $\mathrm{E}(E_{ki}) = m_{1k}$ and variation $\mathrm{D}^2(E_{ki}) = \sigma_k^2$, has got distribution asymptotically normal $\mathrm{N}(m_{1k}, \frac{\sigma_k}{\sqrt{n}})$.

Analysis of toxic substances contained in emission according to the presented proposal is interesting in the aspect that convergence of the statistics $\overline{E}_k$ distribution to the normal distribution $\mathrm{N}(m_{1k}, \frac{\sigma_k}{\sqrt{n}})$ is so quick that it can be used for all $n \geq 4$, so almost always.

When the value σ is known and the distribution $\mathrm{N}(m_{1k}, \frac{\sigma_k}{\sqrt{n}})$ of the statistics $\overline{E}_k$ is considered, it is possible to determine a confidence interval for unknown expected value $m_{k1} = \mathrm{E}(E_k)$ from the formula [3, 8, 10]:

$$P\left\{\overline{e}_k - y_\alpha \frac{\sigma_k}{\sqrt{n}} \leq \mathrm{E}(E_k) \leq \overline{e}_k + y_\alpha \frac{\sigma_k}{\sqrt{n}}\right\} = \beta, \tag{13}$$

where:

β – confidence level,
y_α – standardized variable of normal distribution, meeting the confidence level $\beta = 1 - \alpha$ (α – significance level).

In research practice the value σ is, as a rule, unknown and for its evaluation the following formula [3, 16] is applied:

$$s_k = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e_{ki} - \overline{e}_k)^2}\ .$$

Because the statistics $\overline{E}_k$ has always distribution asymptotically normal $\mathrm{N}(m_{1k}, \frac{\sigma_k}{\sqrt{n}})$, and its convergence to normal distribution $\mathrm{N}(m_1, \sigma)$ is very quick, so practically it can be accepted that analyzed characteristic E_k of emission, being a content of a k toxic substance, has normal distribution $\mathrm{N}(m_{1k}, \sigma_k)$. Hereby, it has to be pointed out that acceptance of assumption that the mentioned characteristic E_k has normal distribution $\mathrm{N}(m_{1k}, \sigma_k)$ is not a limitation in research practice because the statistics $\overline{E}_k$ has always distribution asymptotically normal $\mathrm{N}(m_{1k}, \frac{\sigma_k}{\sqrt{n}})$. Additionally, convergence of this distribution to normal distribution is very quick \[3]. Thus, the random variable $\frac{\overline{E}_k - \mathrm{E}(E_k)}{s_k}\sqrt{n-1}$ has distribution t-Student with $k = n - 1$ degrees of freedom. That means that confidence interval for unknown expected value $\mathrm{E}(E_k)$ of the random variable E_k can be determined from the formula [3, 8]:

$$P\left\{\overline{e}_k - t_{\alpha,n-1}\frac{s_k}{\sqrt{n-1}} \leq \mathrm{E}(E_k) \leq \overline{e}_k + t_{\alpha,n-1}\frac{s_k}{\sqrt{n-1}}\right\} = \beta.$$

4. Final conclusions

Emission of toxic substances, occurring during operation of combustion engines, depends, among others, on their thermal; states. The highest emission of toxic substances is recorded in time of starting the engines. Starting an engine from its cold state, so the state having an ambient temperature, is particularly disadvantageous. Engines in warm state and especially hot state generate smaller amount of toxic substances. Therefore, it is significant to know the probabilities of starting engines from their particular states. For determining the probabilities it has been proposed herein the semi-Markov model of the engine starting three-state process with the following interpretation of the states: cold state (s_1), warm state (s_2), hot state (s_3). It has been shown usability of the model to determine the mass of toxic substances emitted to the surroundings during starting engines. There has also been presented another approach to testing the content of toxic substances in exhaust gases. It consists in considering the mass of any toxic substance as a characteristic of emission, being a random variable. It has also been shown usability of the Lindeberg-Levy's theorem, from which follows that the statistics occurred from results of testing a determined mass of emitted toxic substance is a random variable with distribution asymptotically normal regardless of the character of the random variable which is the tested mass of the mentioned substance. This enables applying the interval estimation for assessing unknown expected value of random variable which can be a mass of any toxic substance existing in exhaust gases.

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